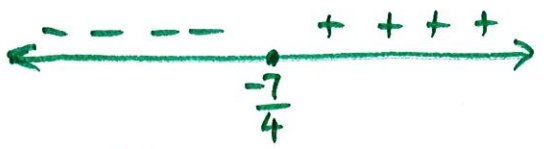
5.3 #7, 9, 13, 15, 16, 21

7. $y = 4x^3 + 21x^2 + 36x - 20$ $\quad y' = 12x^2 + 42x + 36$

$y'' = 24x + 42$ $\quad y'' = 0$ $\quad 0 = 24x + 42$ $\quad x = \frac{-42}{24} = \frac{-7}{4}$



$x = \frac{-7}{4}$ is the point of inflection

$y(x)$ is concave up on $(\frac{-7}{4}, \infty)$ and concave down on $(-\infty, \frac{-7}{4})$

9. $y = 2x^{\frac{1}{5}} + 3$ $\quad y' = \frac{2}{5}x^{\frac{-4}{5}}$ $\quad y'' = \frac{-8}{25}x^{\frac{-9}{5}} = \frac{-8}{25x^{9/5}}$

$y'' \neq 0$ $\quad y''(0) =$ undefined

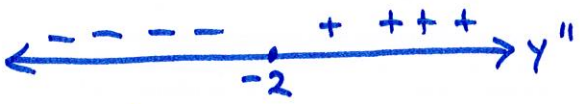
$x = 0$ is a point of inflection

$y(x)$ is concave up on $(-\infty, 0)$ and concave down on $(0, \infty)$

13. $y = xe^x$ $\quad y' = e^x + xe^x$ $\quad y'' = e^x + e^x + xe^x = 2e^x + xe^x$

$y'' = 0$ $\quad 0 = 2e^x + xe^x$ $\quad 0 = e^x(2 + x)$ $\quad x = -2$ $\quad y''(-2) = 0$

$x = -2$ is a point of inflection since $y''(-2) = 0$ and since y'' changes sign at $x = -2$



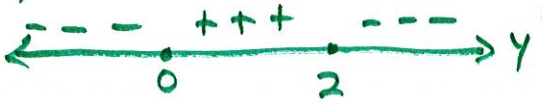
15. $y = \tan^{-1}x$ $\quad y' = \frac{1}{1+x^2}$ $\quad y'' = \frac{0(1+x^2) - (1)(2x)}{(1+x^2)^2} = \frac{-2x}{(1+x^2)^2}$

$y''(0) = 0$

$x = 0$ is a point of inflection since $y''(x)$ changes sign at $x = 0$ and $y''(0) = 0$

16. $y = x^3(4 - x) = 4x^3 - x^4$ $\quad y' = 12x^2 - 4x^3$ $\quad y'' = 24x - 12x^2$

$y'' = 0$ $\quad 0 = 24x - 12x^2$ $\quad 0 = 12x(2 - x)$



$x = 0$ and $x = 2$ are points of inflection since $y''(x)$ changes sign at $x = 0$ and $x = 2$.

21. a. $f' = 0$ at $x \approx -1$ and $x \approx 1$

$f' > 0$ on $(-\infty, -1) \cup (1, \infty)$

$f' < 0$ on $(-1, 1)$

$f'' = 0$ at $x = 0$

$f'' > 0$ on $(0, \infty)$

$f'' < 0$ on $(-\infty, 0)$